# Color and charge breaking minima in the MSSM

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The scalar potential of theories with broken supersymmetry can have a number of local minima characterized by different gauge groups. Symmetry properties of the physical vacuum constrain the parameters of the MSSM. We discuss these constraints, in particular those that result from the vacuum stability with respect to quantum tunneling.

A generic feature of theories with (softly broken) supersymmetry is a scalar potential, $V(\phi)$, that depends on a large number of scalar fields $\phi = (\phi_1, ..., \phi_n)$. For this reason, the scalar potential of the MSSM, unlike that of the Standard Model, may have a number of local minima characterized by different gauge symmetries. In particular, the supersymmetric partners of quarks, $\tilde{Q}_L$ and $\tilde{q_R}$, may have non-zero vev in some minima, where the tri-linear terms $AH_2\tilde{Q}_L\tilde{q_R}$ and $\mu H_1 \tilde{Q}_L \tilde{t}_R$ are large and negative (here $H_{1,2}$ denote Higgs fields and $A$ is the SUSY breaking parameter). These color and charge breaking (CCB) minima may be local, or global, depending on the values of the MSSM parameters. (Of course, there might be directions along which the effective potential is unbounded from below (UFB), in which case all the minima are local.)

Any of these local minima may serve as the ground state for the Universe at present, provided that the lifetime of the metastable state is large in comparison to the age of the Universe. The latter is plausible [1,2] because the tunneling rate in quantum field theory is naturally suppressed by the exponential of a typically large dimensionless number, the saddle point value of the Euclidean action.

Are there any empirical, or general theoretical considerations that could rule out the possibility of the Universe at present being in a long-lived, but metastable, vacuum? Apparently, the answer is no.

Different minima of the scalar potential are characterized by different values of the cosmological constant. However, the cosmological constant problem is just as severe in the stable vacuum as it is in a metastable one. In fact, in a large class of locally supersymmetric Unified theories the cosmological constant can be fine-tuned to be zero in any of the *local* minima, but not in the global minimum [3].

In principle, one can imagine a Gedanken experiment to determine whether the vacuum is true, or false. However, if a metastable vacuum has existed for $\tau_U = 10$ billion years, then any effects of the metastability [4] would be characterized by the scale $< 1/\tau_U \sim 10^{-33}$ eV, beyond any hope of being observable. Sadly, the first direct evidence of the vacuum instability would be a catastrophic event.

The question, however, has more than just eschatological relevance. It is very important scientifically, because ruling out the possibility that the Universe rests in the false vacuum would impose strong constraints on theories of fundamental interactions [1,2,5]. Clearly, only the false vacua whose lifetimes are small in comparison to the age of the Universe are ruled out empirically.

We concentrate on the TeV-scale CCB minima. These generally disappear at temperatures $T \gg 1$ TeV. Therefore, if the temperature of reheating after inflation is 10 TeV or higher, one can assume that the electroweak symmetry is restored. Then at $T = T_c \sim 100$ GeV the electroweak phase transition proceeds from an $SU(3)\times SU(2)\times U(1)$

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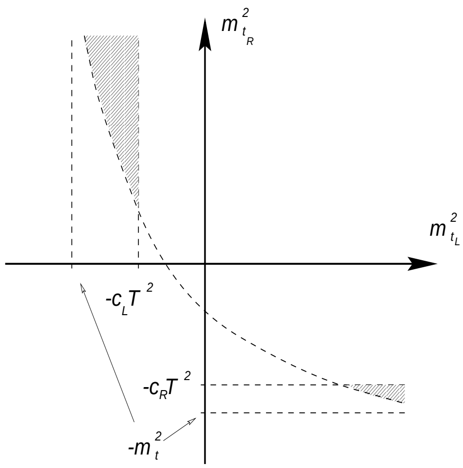


Figure 1. Region of parameters (shaded) which can be ruled out by requiring stability of the $SU(2)\times U(1)$ symmetric minimum above the electroweak transition temperature. Coefficients $c_{L,R}$ are defined in Ref. [2].

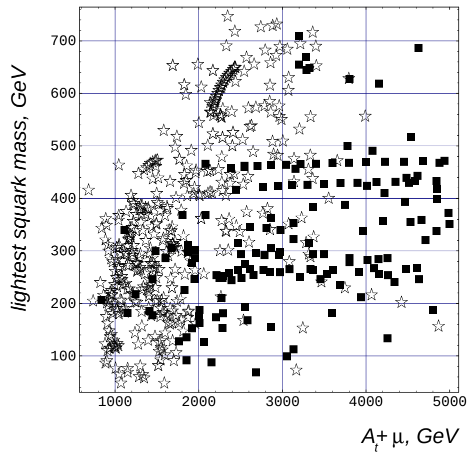


Figure 2. The domains of stability (stars) and instability (boxes) of the false SML vacuum with respect to tunneling into the global CCB minimum. Light top squark and large trilinear couplings generally correspond to a lower and thinner barrier and, thus, higher probability of tunneling.

-symmetric phase to the phase of the broken symmetry. It was shown in [2] that for a wide range of parameters (Fig. 1) this transition favors the Standard Model-like (SML) minimum over the CCB minima.

The probability of tunneling from the SML into a CCB minimum, or a UFB valley, determines the lifetime of a false SML vacuum. Tunneling rate can be evaluated using the semiclassical approximation [6] and is proportional to $\exp(-S[\bar{\phi}])$, where $S[\bar{\phi}]$ is the Euclidean action of the so called "bounce", $\bar{\phi}(x)$, a solution of the classical Euclidean field equations. In practice, however, finding $\bar{\phi}(x)$ numerically is very difficult (or nearly impossible), especially in the case of a potential that depends on more than one scalar field. This is because $\bar{\phi}(x)$ is an unstable solution, as it must be to be a saddle point of the functional $S[\phi]$. An effective alternative to solving the equations of motion is to use the method of Ref. [7]. The idea is to replace the action $S$ with a different functional, $\tilde{S}$, for which the same solution, $\bar{\phi}(x)$, is a minimum, rather than a saddle point. Then $\bar{\phi}(x)$ can be found numerically using a straightforward relaxation technique to minimize $\tilde{S}$.

Another significant simplification comes from the observation that the tunneling rate (in semiclassical approximation) is independent of the physics at the scales large in comparison to $\bar{\phi}(0)$, the escape point. This non-perturbative decoupling [2] of high-energy physics allows one to treat the UFB valleys on the same footing with the very deep CCB minima. Essentially, one can set a cutoff at, *e. g.*, 10 TeV and, as long as the bounce solutions found numerically do not extend beyond this limit, one can justifiably ignore the physics at the higher energy scales.

For a 10 billion year old Universe, the tunneling probability is negligible if $S[\bar{\phi}] > 400$. The most "dangerous" CCB minima, *i. e.* those that to correspond the relatively high tunneling rates, are associated with the third generation squarks. This is because the action of the bounce is proportional [1] to the inverse Yukawa coupling squared.

The requirement of stability of the SML vac-

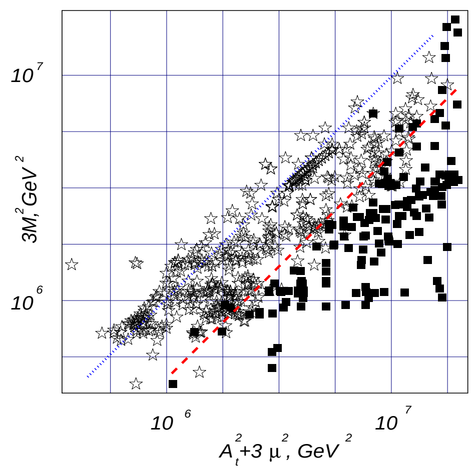


Figure 3. The dotted line represents the empirical criterion for the absence of the global CCB minima: $A_t^2 + 3\mu^2 < 3M^2$, where $M^2 = m^2_{\tilde{t}_L} + m^2_{\tilde{t}_R}$. Taking into account the tunneling rates relaxes this constraint to, roughly, $A_t^2 + 3\mu^2 < 7.5M^2$, shown as the dashed line. The scale is logarithmic.

uum constrains the parameter space of the MSSM [2]. In particular, the trilinear terms in the potential have an upper limit that depends on the quadratic mass terms of squarks (Fig. 2). However, these bounds are not as stringent as they would have been, should one require the SML minimum to be the global minimum of the potential. We found [2] that for a large portion of the parameter space the presence of the global CCB minimum is irrelevant because the time required for the Universe to relax to its lowest energy state exceeds its present age [2]. This is illustrated in Fig. 3.

In summary, the color and charge conserving minimum may not be the global minimum of the MSSM potential. It is possible that the Universe rests in a false vacuum whose lifetime is large in comparison to the present age of the Universe. Under fairly general conditions, the SML vacuum is favored by the thermal evolution of the Universe, even if it does not represent the global minimum. The existence of the CCB minima of the scalar potential results in some important constraints on models with low-energy sypersymmetry. However, the commonly imposed requirement that the SML minimum be global is too strong and may overconstrain the theory.

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